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Very large scale correlations in the galaxy distribution

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Abstract. - We characterize galaxy correlations in the Sloan Digital Sky Survey by measuring several moments of galaxy counts in spheres. We firstly find that the average counts grows as a power-law function of the distance with an exponent $D = 2.1 \pm 0.05$ for $r \in [0.5, 20]$ Mpc/h and $D = 2.8 \pm 0.05$ for $r \in [30, 150]$ Mpc/h. In order to estimate the systematic errors in these measurements we consider the counts variance finding that it shows systematic finite size effects which depend on the samples sizes. We clarify, by making specific tests, that these are due to galaxy long-range correlations extending up to the largest scales of the sample. The analysis of mock galaxy catalogs, generated from cosmological N-body simulations of the standard LCDM model, shows that for $r < 20$ Mpc/h the counts exponent is $D \approx 2.0$, weakly dependent on galaxy luminosity, while $D = 3$ at larger scales. In addition, contrary to the case of the observed galaxy samples, no systematic finite size effects in the counts variance are found at large scales, a result that agrees with the absence of large scale ($r \approx 100$ Mpc/h) correlations in the mock catalogs. We thus conclude that the observed galaxy distribution is characterized by correlations, fluctuations and hence structures, which are larger, both in amplitude and in spatial extension, than those predicted by the standard model LCDM of galaxy formation.

Introduction. – The quantification of large scales galaxy correlations and fluctuations is a central problem in cosmology. The advent of massive redshift surveys has made possible precise measurements of the galaxy two-point correlation function on ten Mpc scale, where power law correlations (in redshift space) have been well established [1–3]. On larger scales the situation is less clear as large density fluctuations on 150 Mpc/h scales were observed [4–11]: these are not obviously compatible with the absence of strong clustering on those scales as predicted by standard models of galaxy formation [12,13]. In addition, in a deep sample of very bright galaxies it was observed [14–16], on 200 Mpc/h scales, an unexpected strong signal with respect to the predictions of the standard model; recently an excess of clustering was also found by [17,18].

In order to characterize the large scale galaxy correlations it was measured, in the Sloan Digital Sky Survey (SDSS) [19,20], the average density $\overline{n(r)}$: this was shown to present a scaling behavior as $\overline{n(r)} \sim r^{-1}$ for $r < 20$ Mpc/h [3,21]. However, while some authors [21] noticed that around 70 Mpc/h it occurs a transition toward uniformity (i.e. $\overline{n(r)} \sim const.$), others concluded that $\overline{n(r)} \sim r^{-0.2}$ for 20 Mpc/h $< r <$ 80 Mpc/h [22]. Furthermore this latter behavior was shown to be associated to a scaling of the variance slower than for a uniform distribution and to a probability density function (PDF) of galaxy counts in spheres well fitted by a Gumbel distribution (i.e., different from a simple Gaussian PDF) [22]. It was then found that the PDF, in different spatial volumes, present systematic differences at large enough scales which, by making specific tests, were interpreted as due to large scale structures [23,24]. However, the considered tests used non-overlapping sub-samples covering different redshift ranges, so that a physical or observational scale dependent systematic effect can, at least partially, affect the observed behaviors.

For instance, it was noticed by that number density of bright galaxies increases by a factor ≈ 3 as redshift increases from $z = 0$ to $z = 0.3$ [25], and to explain these observations a significant evolution in the luminosity and/or number density of galaxies at redshifts $z < 0.3$ was proposed. In this context evolution can be parameterized as a redshift dependent correction [26]. However, by performing several tests to determine the possible effect of evolu-

tion on the SDSS data, it was concluded [24] that while the evolution may change the galaxy redshift counts behavior as a function of redshift, but it is unlikely that it can produce the large amplitude fluctuations of large spatial extension.

In this letter, by considering SDSS equal volume samples covering the same distance scales, we are able to disentangle finite-size effects due to large scale correlations from redshift dependent systematic effects. This allows us to confirm the results by [22,24], to extend them to larger scales, i.e. $r \approx 150$ Mpc/h, and to clarify several other properties of the large scale galaxy distribution.

The Data. – We have constructed several sub-samples of the main-galaxy (MG) sample of the spectroscopic catalog SDSS-DR7 [27,28]. The selection criteria used to construct the volume limited (VL) [1] samples are (see [24] for more details): we selected galaxies from the MG sample with redshift $z \in [10^{-4}, 0.3]$ with redshift confidence $z_{conf} \geq 0.35$, without significant redshift determination errors and with apparent magnitude $m_r < 17.77$. In order to avoid the irregular edges of the survey boundaries, and to consider a simple sky area, we have constructed a sample (R_1) limited, in the internal angular coordinates of the survey [24], by $\eta \in [-33, 5°, 36.0°]$ and $\lambda \in [-48°, 51.5°]$. We also consider in what follows two half samples of equal volume are limited by (R_2) $\lambda \in [-48°, 0]$ and (R_3) $\lambda \in [0°, 51.5°]$.

The survey is known to have a small angular incompleteness: indeed, in average, about the 5% of the target galaxies have not measured redshift [28]. In general, there is not a free of a-priori assumptions procedure, to correct for such an incompleteness. Indeed, given that the detailed information of the real galaxy distribution is unknown one has to make some assumptions on the statistical properties of such a distribution (see e.g. [1]). As we do not want to use such ad-hoc assumptions, as we employ statistical methods that aim to tests some of the most common assumptions in the analysis of galaxy correlations [30], we have adopted the following strategy. From the one hand we have considered an angular region which does not include the survey edges where completeness varies mostly. From the other hand we have done a few tests to control the effect of completeness on the correlation analysis, and in particular, as we discuss below, we have focused to test the *stability* the results. Note that the incompleteness due to fiber collisions can be neglected in measurements of large-scale (i.e., $r > 10$ Mpc/h) galaxy correlations as this effects is only relevant very small scales (see [24] and references therein).

In order to construct VL samples (see Tab.1) we computed the metric distances R using the standard cosmo-

VL sample	R_{min}	R_{max}	M_{min}	M_{max}	N_p
VL1	50	200	-18.9	-21.1	73811
VL2	125	400	-20.5	-22.2	129975
VL3	200	600	-21.6	-22.8	51698

Table 1: Main properties of the SDSS VL samples : R_{min}, R_{max} (in Mpc/h) are the chosen limits for the metric distance; M_{min}, M_{max} define the interval for the absolute magnitude in each sample and N_p is the number of galaxies in the sample.

logical parameters, i.e., $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. Absolute magnitudes M are computed using Petrosian apparent magnitudes in the m_r filter corrected for Galactic absorption and applying standard K-corrections [29].

Statistical methods. – We consider the number of galaxies $N_i(r)$ within radius r around the ith galaxy: this quantity differs for each galaxy and hence we consider it as a random variable characterizing its statistical properties. The average over an ensemble of realizations, $\langle N(r) \rangle$, can be estimated by the volume average (assuming ergodicity)[2]

$$\overline{N(r)} = \frac{1}{M(r)} \sum_{i=1}^{M(r)} N_i(r) . \quad (1)$$

Note that the number of points contributing to the average (Eq.1), $M(r)$, depends on the scale r as an effect of the requirement that the spheres must be fully included in the sample's boundaries [24]. To estimate the typical fluctuations of the random variable $N_i(r)$ we may consider the conditional variance $\Sigma^2(r) = \langle N^2(r) \rangle - \langle N(r) \rangle^2$. In general [30], this can be written as the sum of two contributions: $\Sigma^2(r) = \Sigma_i^2(r) + \Sigma_p^2(r)$, where $\Sigma_i(r)$ is the intrinsic part, due to correlations, and $\Sigma_p^2(r) = \langle N(r) \rangle$, due to Poisson noise. The normalized variance is defined as $\sigma^2(r) = \Sigma^2(r)\langle N(r) \rangle^{-2} = \sigma_i^2(r) + \sigma_p^2(r)$, where its intrinsic part can be estimated by

$$\overline{\sigma_i^2(r)} = \frac{1}{\overline{N(r)}^2} \times \frac{\sum_{i=1}^{M(r)}(N_i - \overline{N(r)})^2}{M(r)} - \frac{1}{\overline{N(r)}} . \quad (2)$$

Results. – We have measured the conditional average density, $\overline{n(r)} = \overline{N(r)}/V(r)$ where $V(r) = 4/3\pi r^3$, in the different SDSS VL samples (see Fig.1). We find that: (i) at small scales, $r \in [0.5, 20]$ Mpc/h $\overline{n(r)} \propto r^{-\gamma}$ with $\gamma = 0.88 \pm 0.05$ (i.e. $\overline{N(r)} \propto r^D$, with $D = 3 - \gamma = 2.12 \pm 0.05$). The error of the exponent has been derived by measuring the scattering in the different samples.

(ii) At larger scales, i.e. $r > 30$ Mpc/h and up to ≈ 150 Mpc/h in the deepest sample, the exponent is $\gamma = 0.2 \pm 0.05$ (i.e. $D = 2.8 \pm 0.05$). The amplitude of the conditional average density in the different VL samples is in principle fixed by a luminosity factor that depends on

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[1] The prescription used to define a volume limited sample takes into account the luminosity selection effects that necessarily affect a survey, i.e. that intrinsically faint galaxies are observed only if they are located close to the observer while intrinsically bright galaxies are observed both if they are nearby to and faraway from the observer.

[2] The symbols $\langle ... \rangle$ (---) stands for the ensemble (volume) average performed with the condition that the sphere center coincides with a point of the distribution, i.e. it is a conditional average [30].
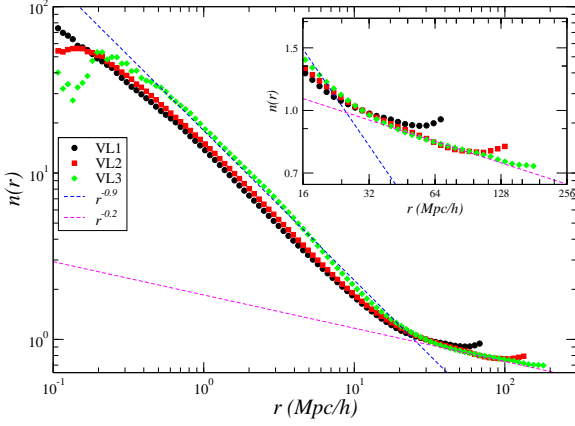
Fig. 1: Galaxy average density normalized to its value at 30 Mpc/h, for the different SDSS samples. The exponent γ reported in the labels corresponds to the best fit in the range [0.5,20] Mpc/h.

Fig. 2: Normalized conditional variance (Eq.2) in the different samples: circles represent the intrinsic variance $\overline{\sigma_i^2(r)}$, while the solid lines the total variance $\overline{\sigma^2(r)}$ (see Eq.2). The dashed line has a slope $1/r^3$ as for pure Poisson noise, a behavior that describes only the very small scales.

the absolute luminosity of the galaxies therein included [24]. However, in a finite sample, as long as correlations extend on scales of the order of the sample size, the amplitude of $\overline{n(r)}$ also depends on the particular structures present in that specific volume. Therefore the galaxy counts normalization in samples of different sizes, covering different space volumes and including galaxies of different luminosity depends on systematic effects which become negligible only for very large sample sizes. For this reason in Fig.1, by choosing an arbitrary normalization, we have plotted $\overline{n(r)}/\overline{n(r_*)}$ where $r_* = 30$ Mpc/h. On the other hand, note that the *exponent* of $\overline{n(r)}$ does not show variations larger than the estimated error bar (see below) in the different samples.

The main result from this analysis is that the *slopes* of the galaxy average density, both at small and large scales, show a very good agreement in the different samples. When the radius r approaches the size of the largest sphere included in each sample there are systematic effects, as shown by the fact that the large scale tail of $\overline{n(r)}$ (where $M(r)$ in Eq.1 becomes rapidly very small) grows at different scales in the different samples.

In order to quantify fluctuations affecting the large scale (i.e. $r > 10$ Mpc/h) behavior of $\overline{n(r)}$ it is necessary to estimate statistical errors. For instance, one may simply compute them as $\sqrt{\Sigma^2(r)V^{-2}(r)M^{-1}(r)}$: however, in this way one underestimates the true errors because large scale correlations can break the Central Limit Theorem and thus the different determinations are not independent [30] (note that such error bars would not be even visible in the plot in Fig.1). A possible evaluation of the scattering of $\overline{n(r)}$ can be performed by measuring sample-to-sample variations [15]. To this aim, we calculate thus Eq.2 in the different samples (see Fig.2) finding that at small scales results are very similar while at large scales a clear difference is detected.

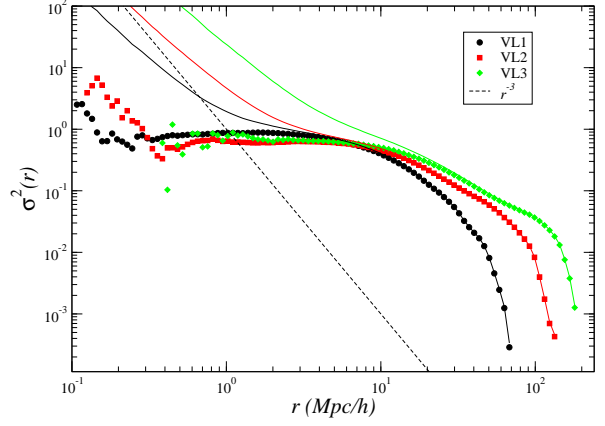Interestingly the differences in $\overline{\sigma_i^2(r)}$ occur at a scales

that grows with sample's sizes. The determination in the smallest sample, i.e. VL1, shows a marked large scale difference with respect to those in VL2 and VL3, which instead present a more similar behavior everywhere but the very large scale tail (i.e. $r \approx 100$ Mpc/h). Therefore a possible explanation of this large scale behavior is that it is due to a finite size effect, i.e. that the scale at which the abrupt decay of $\overline{\sigma_i^2(r)}$ occurs in the different samples, depends on the their sizes. To show that this is the case we have considered the behavior of $\overline{\sigma_i^2(r)}$ in sub-volumes of the VL2 and VL3 samples limited to a depth (i.e., R_{max}) smaller than the ones reported in Tab.1. In this way we are able to clearly single out the finite size effect (see Fig.3).

We performed another test by cutting the sample into two subsamples of equal volume (i.e., limited in angles by the regions R_2 and R_3) and determining $\overline{n(r)}$ and $\overline{\sigma_i^2(r)}$ in each of them (see Fig.4). In the case of VL3 the determinations of both the average density and the variance in two subsamples agree better than for VL1: this corroborates the result, obtained also with the previous tests, that systematic finite size effects become weaker in samples with larger volumes [22,24,31]. This is in agreement with the larger dimension observed at large scale, which also implies less wild fluctuations [30].

From these behaviors we may conclude that both $\overline{n(r)}$ and $\overline{\sigma^2(r)}$ converge to a well-defined value only for scales $r < r_s$ smaller than the radius of the largest sphere included in the sample. We estimate that for VL1 $r_s \approx 50$ Mpc/h, for VL2 $r_s \approx 100$ Mpc/h and for VL3 $r_s \approx 150$ Mpc/h. Beyond these scales the behaviors in the two half samples systematically differ: this is in agreement with the results by [8, 9, 22–24, 31] where the whole behavior of the PDF of $N_i(r)$ was considered.

As mentioned above it is interesting to consider the effect of the small (i.e. $\sim 5\%$) angular incompleteness on the measurements of galaxy correlations. For instance, there
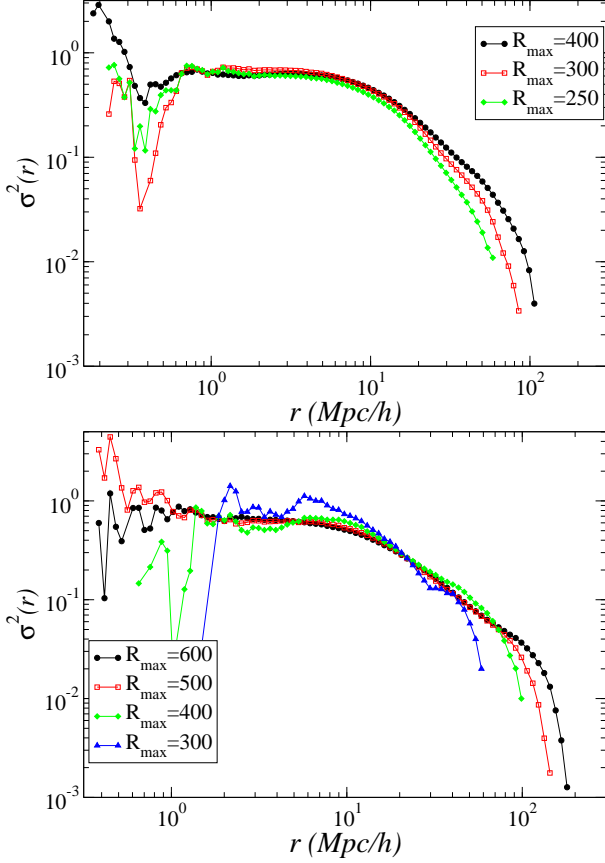
Fig. 3: Normalized conditional variance in subsamples of VL2 (upper panel) respectively with $R_{max} = 250, 300, 400$ Mpc/h and VL3 (bottom panel) with $R_{max} = 300, 400, 500, 600$ Mpc/h.

Fig. 4: Normalized conditional variance in the three regions of VL1 (upper panel) and VL3 (bottom panel).

are some small sky regions where, due to the presence of bright stars, galaxies have not been observed. Each bright star corresponds to a circular hole where galaxies cannot be observed. The radius of the circle around each bright star depends on the star apparent magnitude [29]. An upper limit to such a size is $\theta = 3$ arsec, corresponding to very bright stars [29]. A simple way to test for effect of such holes is the following.

We distribute randomly holes of size θ with a surface density of 50 per square degree (roughly corresponding the surface density of bright stars [32]). Galaxies which are placed inside one of the holes are cut from the resulting sample. In such a way we artificially have taken away from the sample, in a correlated way at small angles, about $\sim 10\%$ of the galaxies (i.e. about the double than the angular incompleteness of the catalog). The question is whether the large scales (i.e., $r > 1$ Mpc/h) correlation properties are affected by such an incompleteness. Results are shown in Fig.5: one may note that, a part an obvious 10% shift in the amplitude of the average conditional density, no scale-dependent changes are manifested. Thus we can confidently conclude that incompleteness effects do not play a major role in the results of the correlation anal-
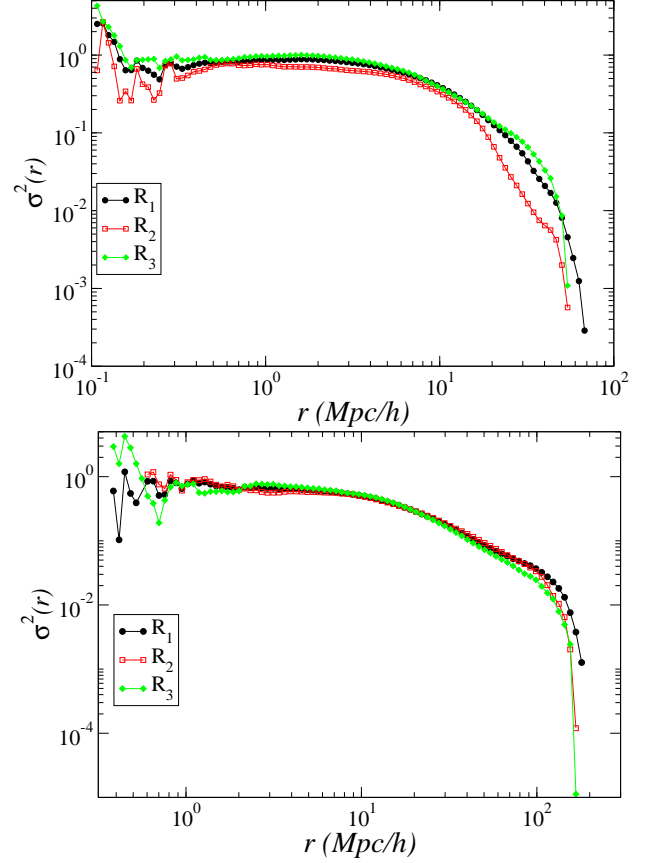
ysis. We refer to a forthcoming work for a more complete series of tests on the angular incompleteness of this survey.

Mock galaxy catalogs. – We have repeated the same analysis in some mock galaxy samples. These are constructed from cosmological N-body simulations of the standard LCDM model [33,34], by applying the same cuts in absolute magnitude and distance as those reported in Tab.1 and by computing the redshift positions (i.e., simply applying the corrections to the redshift due to the peculiar velocities along the line of sight). The conditional average density (see Fig.6) shows (i) a slope $\gamma \approx 1$ for $r \in [0.5, 20]$Mpc/h that is weakly dependent on the average luminosity of the galaxies, and (ii) a well defined crossover to uniformity, i.e. $\gamma = 0$, at ~ 30 Mpc/h. Both features are different from the real SDSS samples. Correspondingly to the crossover toward uniformity (absence of large scales strong correlations), the conditional variance (Fig.7) does not show a finite size dependence, contrary to what occurs for the real samples (Fig.2). In this case, the stability of the normalized variance in the different samples is shown by diving the samples into two parts and by comparing the behaviors: because of the lack of large scale correlations, no systematic differences are found between the two subsamples.
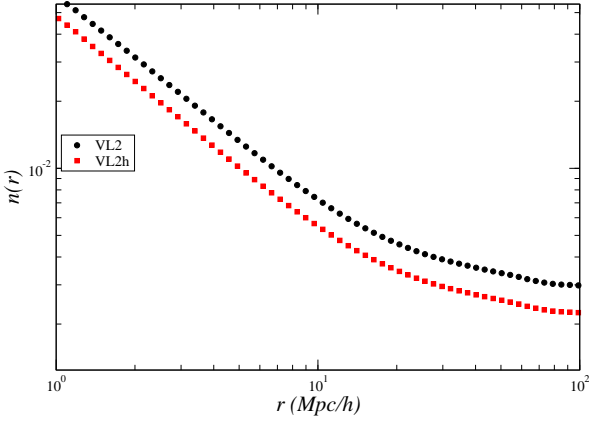
Fig. 5: Conditional average density for the full sample VL2 and for a modified version of it (VL2h) where ∼ 10% of the galaxies have been cut in a correlated way (see text): a part an obvious 10% shift in the amplitude of the average conditional density, not scale-dependent changes are manifested.
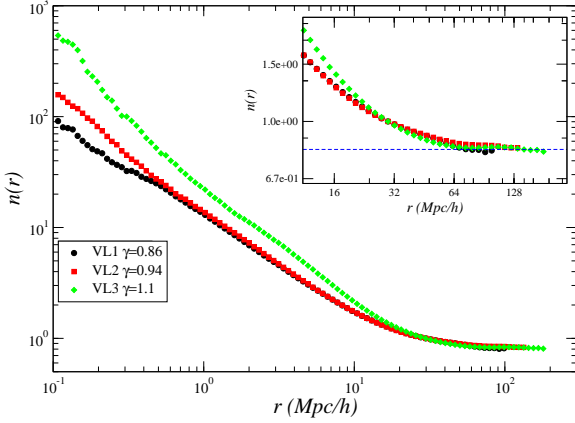

Fig. 6: Conditional average density, normalized to its value at 30 Mpc/h, for the different samples of the mock galaxy catalog. The exponent γ reported in the labels is the best fit in the range [0.5,20] Mpc/h
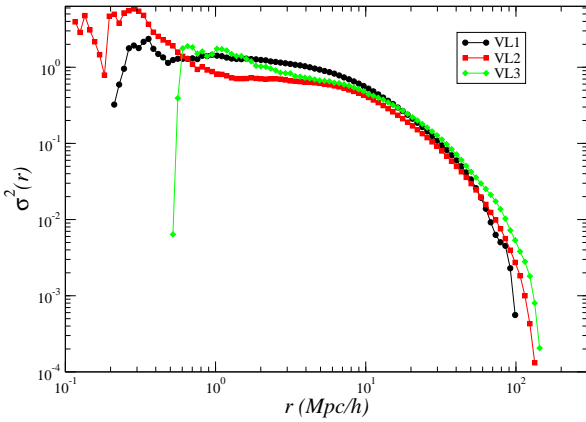

Fig. 7: Normalized conditional variance for the different samples of the mock galaxy catalog. No finite size effects are present in this case.

Note that a given mock galaxy sample is constructed from the underlying dark matter particles distribution, by assuming a certain prescription to identify galaxies. Such a prescription is generally based on a physical mechanism which links the local density of dark matter particles to the probability to form a mock galaxy [34]. In principle, one can introduce different physically motived prescriptions to relate dark to visible matter distributions in cosmological simulations. The question is then whether a different prescription can substantially change the resulting mock galaxies correlation properties, possibly giving rise to a better agreement with observations. We do not explore this question here and we cannot exclude that a better agreement could be obtained with a different bias prescription. However we note that, given that large-scale correlations are not present in the underlying dark matter distribution, as this is what theoretical models predicts on scales of the order of ∼ 100 Mpc/h [13,35], the only way to introduce such correlations is via a bias mechanism: biasing should correspond to a large scale correlated selection. This implies the biasing mechanism to be non-local, i.e. a completely different one from what is generally considered to be a physically plausible prescription.

Discussion. – In summary, we have analyzed the scaling properties of the galaxy distribution in the SDSS-DR7 samples. We found that at small scales $r \in [0.5, 20]$ Mpc/h the galaxy (conditional) average density decays as a power-law function of distance, $\overline{n(r)} \propto r^{-\gamma}$ with an exponent $\gamma \approx 0.9$ while at large scales, i.e. $r \in [30, 150]$ Mpc/h, it decays slower: $\gamma \approx 0.2$. The analysis of the variance (Eq.2) allows to clearly single out a finite size effect, that can be simply understood as due galaxy correlations extending up to the largest scales of the considered samples. These behaviors differ from those found in mock galaxy samples where: (i) at small scales the exponent, $\gamma \approx 1$, slightly depends on galaxy luminosity, and (ii) at large scales, i.e. $r > 30$ Mpc/h, a well defined crossover to uniformity (i.e. $\gamma = 0$) is found. In addition, no finite size effects are detected in the large scale behavior of the normalized counts variance.

The detection of very large scale galaxy correlations and of finite-size effects, allow us to conclude that the observed galaxy distribution is more correlated, fluctuating and thus clustered, than the one predicted by the standard LCDM model of galaxy formation through cosmological N-body simulations [33,34]. These findings are in agreement with the results obtained in the same samples, although at smaller scales $r < 100$ Mpc/h, through different tests [10,22–24,31], with the results obtained in the 2 degree field redshift survey [4–9] and other surveys [14–18]. Note that the results obtained by [21] are also compatible with a slow decay of $\overline{n(r)}$ at large scales.

Because of the large-scale scaling of the galaxy average density we conclude that any statistical quantity normalized to the estimation of the sample average density (e.g., the standard two-point correlation function) is biased by

finite size effects [30]. This implies that the volume of current galaxy samples is not yet large enough to measure the standard two-point correlation function at ~ 100 Mpc/h.

From a theoretical point of view, the main challenge of our results concerns the way in which the large scale universe is modeled. Indeed the deviation from a pure statistically homogeneous and isotropic, and spatially uniform density field [31] imply the consideration of the inhomogeneities effects on the dynamics of the universe and the deviations that can possibly be introduced with respect to the simple FRW models [36–40].

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